ISEL

Análise Matemática I

Departamento de Engenharia Mecânica

2° Semestre 2005/2006

Ficha 4

1. Calcule o valor médio de cada uma das seguintes funções, e interprete geometricamente o resultado.

   (a) $f(x) = \sin^2(x)$ no intervalo $[0, \pi]$;

   (b) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ no intervalo $\left[0, \dfrac{1}{\sqrt{2}}\right]$.

2. Calcule o valor médio da função $f(x) = \mathrm{e}^{-x}\sin(x)$ no intervalo $[0, 2\pi]$. Mostre que existe $y \in \,]0, 2\pi[$ tal que $f(y) = \dfrac{1}{4\pi}(1 - \mathrm{e}^{-2\pi})$. Justifique.

3. Calcule:

   (a) $\displaystyle\int_1^2 \frac{4x^2+3x+2}{x(x^2+x+1)}\,\mathrm{d}x$.

   (b) $\displaystyle\int_{\mathrm{e}}^{\mathrm{e}^2} \frac{\sqrt{3+\ln(x^2)}}{x}\,\mathrm{d}x$.

   (c) $\displaystyle\int_0^1 \frac{\mathrm{e}^{2x}}{1+\mathrm{e}^x}\,\mathrm{d}x$.

4. Determine a função $f : \mathbb{R}_0^+ \to \mathbb{R}$, não nula e diferenciável, que verifica a condição

$$[f(x)]^2 = \int_0^{x^2} \frac{\sqrt{t}}{2+t} f(\sqrt{t})\,\mathrm{d}t.$$

5. Considere a função definida por $F(x) = \displaystyle\int_{\arctan(x)}^{\ln(1+x^2)} \frac{\sin(t)}{t}\,\mathrm{d}t$. Calcule, justificando, $F'(x)$.

6. Determine a equação da recta tangente ao gráfico de $f(x) = \displaystyle\int_1^{1+x^2} \frac{1+t}{t}\,\mathrm{d}t$ no ponto $x = -1$.

7. Considere as funções $F(x) = \displaystyle\int_1^x \ln(t^2)\,\mathrm{d}t$ e $G(x) = \displaystyle\int_1^{x^4} \ln(\sqrt{t})\,\mathrm{d}t$. Mostre que

$$F'(x) + G'(x) = F'(x)(1+4x^3).$$

8. Calcule $\displaystyle\lim_{x\to 0} \frac{\displaystyle\int_{x^2}^{x^3} \sin^2(t)t^2\,\mathrm{d}t}{x^3}$.

9. Calcule:

   (a) $\displaystyle\int_0^{+\infty} x\mathrm{e}^{-x}\,\mathrm{d}x$

   (b) $\displaystyle\int_0^{\frac{\pi}{2}} \tan(3x)\sec^4(3x)\,\mathrm{d}x$

   (c) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{x^2+2x+7}\,\mathrm{d}x$

   (d) $\displaystyle\int_0^{+\infty} \frac{\sqrt{\arctan(2x)}}{1+4x^2}\,\mathrm{d}x$

   (e) $\displaystyle\int_{\mathrm{e}}^{+\infty} \frac{\ln(x)}{x^2}\,\mathrm{d}x$

10. Calcule a área da região limitada por:

    (a) $y = \cos(x)$, $y = \sin(x)$, $x = \dfrac{\pi}{4}$ e $\dfrac{5\pi}{4}$;
    (b) $y^2 = 2 - x$ e $y = x$;
    (c) $y = \mathrm{e}^x$, $y = (x-1)^2$ e $x = 1$;
    (d) $y = -x^2 + 4$ e $y = (x-2)^2$;
    (e) $y = 3^{-2x}$, $y = \sqrt{1-x^2}$ e $x = -1$.

Soluções.

1. (a) $\dfrac{1}{2}$; (b) $\dfrac{\sqrt{2}\pi}{4}$.
2. $\dfrac{1-\mathrm{e}^{-2\pi}}{4\pi}$.
3. (a) $\ln\left(\dfrac{28}{3}\right)$; (b) $\dfrac{1}{3}\left(\sqrt{7^3} - \sqrt{5^3}\right)$; (c) $\mathrm{e} - 1 + \ln\left(\dfrac{2}{1+\mathrm{e}}\right)$.
4. $f(x) = x - \sqrt{2}\arctan(\frac{\sqrt{2}x}{2})$.
5. $\dfrac{\sin(\ln(1+x^2))}{\ln(1+x^2)}\dfrac{2x}{1+x^2} - \dfrac{\sin(\arctan(x))}{\arctan(x)}\dfrac{1}{1+x^2}$.
6. $y = -3(x+1) + 1 + 1\ln(2)$.
8. 0.
9. (a) 1; (b) $+\infty$; (c) $\dfrac{\sqrt{6}\pi}{6}$; (d) $\dfrac{1}{3}\sqrt{\dfrac{\pi^3}{8}}$; (e) $\dfrac{2}{\mathrm{e}}$.
10. (a) $2\sqrt{2}$; (b) $\dfrac{9}{2}$; (c) $\mathrm{e} - \dfrac{2}{3}$; (d) $\dfrac{8}{3}$; (e) $\dfrac{\pi}{4} + \dfrac{1}{2\mathrm{e}^2} - \dfrac{1}{2}$.